Generalised co-ordinates

To describe the configuration of a system, we select the smallest possible number of variables. These are called the generalised co-ordinates of the system.

A set of generalised co-ordinates is any set of co-ordinates which describe the configuration.

For example, in dealing with rigid body motion we specify three cartesian co-ordinates to locate the centre of mass with respect to an external origin and three angle co-ordinates relative to origin at the centre of mass. Thus, the generalised co-ordinates should all be chosen the conventional orthogonal position co-ordinates or all may be angle co-ordinates.

Notation for generalised co-ordinates:

Generalised co-ordinates are designated by a letter q with numerical subscripts; $q_1, q_2, \dots q_n$ represent a set of n generalised co-ordinates. Or, by a letter subscript to q

and specifying within brackets the numerical values that the letter subscript is allowed to take, e.g. q_j ($j = 1, 2, \cdots n$).

Some examples of mechanical systems with the generalised co-ordinates are given below:

System	Generalised Co-ordinates
1. Simple pendulum	θ; the angle which the pendulum makes with vertical line through point of suspension
2. Fly-wheel	θ; the angle b/w a definite radius of the fly wheel and fixed line perpendicular to the axis
3. Particles on the surface of a sphere	θ, φ: the usual polar angle of a point on the sphere.
4. Beads on an abacus	x; the cartesian co-ordinate along the horizontal axis

Constraints

A constrained motion is a motion which cannot proceed arbitrarily in any manner. Particle motion is restricted to occur only, for example, along some specified path, or on a surface (plane or curved) arbitrarily oriented in space. Motion along a specified path is the simplest example of a constrained motion.

Thus imposing constraints on a mechanical system is to simplify the mathematical description. Condition imposed on the system by the constraints can, in most cases be written down mathematically as a relation satisfied by the co-ordinates of the particle at any time.

This is the way in which constraints reduce the number of co-ordinates needed to specify the configuration of a system.

Ex. 1: Let us consider the motion of a simple pendulum confined to move in the vertical plane. We would need only two co-ordinates to locate the position of the bob in motion. However, motion of the bob is not free but takes place under a constraint that the distance l of the bob is to remain the same from O all the time. This condition imposed by the constraint can be expressed in the form of an equation either between x and y or r and θ:

$$x^2+y^2 = l^2$$

$$\text{or,} \quad r = l \quad \text{——— (1)}$$

Ex.2: Let us take up another example. A particle moving in space requires three co-ordinates to determine its position at any instant. If we restrict its movement on the surface of a sphere, there exists a relation between these co-ordinates.

$$x^2+y^2+z^2 = a^2$$

$$\text{or,} \quad r = a$$

in which a is the radius of the sphere.